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On the propagation of elasto-thermodiffusive surface waves in heat-conducting materials

J.N. Sharma[a,*], Y.D. Sharma[a], P.K. Sharma[b]

[a]*Department of Applied Sciences, National Institute of Technology, Hamirpur 177 005, India*
[b]*Department of Mathematics, SL Bawa Dayanand Anglo Vedic College, Batala 143505, India*

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Abstract

The present paper deals with the study of the propagation of Rayleigh surface waves in homogeneous isotropic, thermodiffusive elastic half-space. After developing the formal solution of the model, the secular equations for stress free, thermally insulated or isothermal, and isoconcentrated boundary conditions of the half-space have been obtained. The secular equations have been solved by using irreducible Cardano's method with the help of DeMoivre's theorem in order to obtain phase velocity and attenuation coefficient of waves under consideration. The motion of the surface particles during the Rayleigh surface wave propagation is also discussed and found to be elliptical in general. The inclinations of wave normal with the major axis of the elliptical path of a typical particle have also been computed. Finally, the numerically simulated results regarding phase velocity, attenuation coefficient, specific loss and thermo-mechanical coupling factors of thermoelastic diffusive waves have been obtained and presented graphically. Some very interesting and useful characteristics of surface acoustic waves have been obtained, which may help in improving the fabrication quality of optical and electronic devices in addition to construction and design of materials such as semiconductors and composite structures. Therefore, this work finds applications in the geophysics and electronics industry.

1. Introduction

Diffusion is the spontaneous movement of matter (particles), from a region of high concentration to low concentration. Diffusion occurs in response to a concentration gradient expressed as the change in concentration due to a change in position. The example of diffusion is heat transport or momentum transport. The net flux of a transported quantity (atoms, energy, or electrons) is equal to a physical property (diffusivity, thermal conductivity, and electrical conductivity) multiplied by a gradient (concentration, thermal, and electric field gradient). Nowadays, there is a great deal of interest in the study of this phenomenon due to its applications in the geophysics and electronic industry. Technologies based on diffusion waves have improved biomedical diagnostics and the fabrication of optical and electronic devices. In integrated circuit fabrication,

*Corresponding author. Tel.: +91 1972 23296; fax: +91 1972 23834.
E-mail addresses: jns@nitham.ac.in (J.N. Sharma), yds@nitham.ac.in (Y.D. Sharma), pars1sharma@yahoo.com (P.K. Sharma).

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diffusion is used to introduce "dopants" in controlled amounts into the semiconductor substance. In particular, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors, and the source/drain regions in MOS transistors and dope polysilicon gates in MOS transistors. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases (e.g. xenon) and other light isotopes (e.g. carbon) for research purposes. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction. However, there is a certain degree of coupling with temperature and thermal gradients as temperature speeds up the diffusion process. The thermoelastic diffusion in elastic solids is due to coupling of the fields of temperature, mass diffusion (MD) and that of strain in addition to heat and mass exchange with the environment. Angstrom [1] was the first to publish an experimental and theoretical study of diffusion waves. In this pioneering work, he calculated the thermal diffusivity of solids as measured by periodically heating a long bar and then detecting the alternating temperature field at a point in the bar some distance away from the heat source. Nowacki [2–5] developed the theory of thermoelastic diffusion by using a coupled thermoelastic model. Dudzviak and Kowalski [6] and Olesiak and Pyryev [7], respectively, discussed the theory of thermo diffusion and coupled quasi-stationary problems of thermal diffusion for an elastic cylinder. They studied the influence of cross effects arising from the coupling of the fields of temperature, MD, and strain due to which the thermal excitation results in additional mass concentration and this generates additional fields of temperature.

During the last three decades, nonclassical theories of thermoelasticity called "Generalized thermo-elasticity" have been developed in order to remove the paradox of physically impossible phenomenon of infinite velocity of thermal signals in the conventional coupled thermoelasticity. Lord and Shulman [8] formulated a generalized theory of thermoelasticity with one thermal relaxation time, which involves a hyperbolic equation of heat transportation, by incorporating a flux-rate term into Fourier's law of heat conduction. Green and Lindsay [9] developed a temperature-rate-dependent thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier law of heat conduction, when the body under consideration has a center of symmetry. The Lord and Shulman [8] theory of generalized thermoelasticity was further extended to homogeneous anisotropic heat conducting materials by Dhaliwal and Sherief [10]. All these theories predict a finite speed of heat propagation. Chandrasekharaiah [11] referred to this wave-like thermal disturbance as "second sound". A survey article of various representative theories in the range of a generalized thermoelasticity was brought out by Hetnarski and Ignaczak [12]. The propagation of thermoelastic Rayleigh waves has been discussed in detail by Nayfeh and Nasser [13] in the context of generalized theory of thermoelasticity developed by Lord and Shulman [8].

The recent development of generalized theory of thermoelastic diffusion by Sherief et al. [14] allows the finite speed of propagation of waves and it provides a chance to study wave propagation in such interesting media. They derived governing equations for generalized thermo diffusion in elastic solids and also proved variational principles and reciprocity theorems for these equations. The uniqueness of solution for these equations under suitable conditions is also established. Singh [15,16] investigated the reflection of P and SV waves from the free surface of elastic solids with generalized thermo diffusion. Sharma [17] studied the propagation of plane harmonic generalized thermoelastic diffusive waves in heat-conducting solids. It is found that there are three longitudinal waves, namely, elastodiffusive (ED), MD-mode) and thermodiffusive (TD-mode) which are possible to propagate in such solids in addition to decoupled transverse waves. According to Achenbach [18], unlike the hyperbolic solution, the classical solution shows no distinct wave front and temperature increase start initially. However, the difference in the predicted temperature between the two theories is small and is apparent for very small time scales (of the order of 100 ps). These time scales are large enough for the solution given by both theories to be numerically undistinguishable. Consequently, the selection of the theory for the time scale of interest can be done for convenience with no practical effect on the calculated results.

Keeping in the mind the above applications of TD processes, the propagation of elasto-TD waves have been investigated in this paper. The secular equation for Rayleigh-type surface waves have been obtained in simplest form and in closed mathematical conditions. The phase velocity and attenuation coefficient of wave propagation have been computed from the secular equations by using irreducible Cardano's method with the help of DeMoivre's theorem and the functional iteration technique. Although there is a precise numerical technique to solve the Rayleigh wave frequency equation, which can avoid the missing root developed by Gao et al. [19], we have used a hybrid of direct and iterative techniques to solve secular equation in which the scope of root missing is negligible. The specific loss and thermo-mechanical coupling factors have also been determined. The surface displacement components, temperature change, and mass concentration of the surface particles have been obtained during Rayleigh wave propagation in addition to the discussion of surface particle motion. The results have also been computed numerically and presented graphically.

2. Formulation of the problem

We consider a homogeneous isotropic, thermo diffusive, elastic half-space initially at uniform temperature T_0 and concentration C_0. We take any point O as the origin of the rectangular Cartesian coordinate system and the z-axis pointing vertically downward in the half-space, which is thus represented by $z \geqslant 0$. We take the x-axis along the direction of wave propagation in such a way that all the particles on a line parallel to the y-axis are equally displaced. Then all the field quantities are independent of y. The surface $z = 0$ is assumed to be stress free, thermally insulated or isothermal and isoconcentrated. We consider the waves of small amplitudes and assume that disturbances are confined to the neighborhood of the free surface ($z = 0$). The basic governing equations in the context of the linear theory of generalized TD elastic solids, for the displacement vector $\vec{u}(x, z, t) = (u, 0, w)$, temperature change $T(x, z, t)$ and mass concentration $C(x, z, t)$ in the absence of body forces and heat sources are given by Sherief et al. [14] as follows:

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\nabla(\nabla \cdot \vec{u}) - \beta_1\nabla T - \beta_2\nabla C = \rho\ddot{\vec{u}} \tag{1}$$

$$K\nabla^2 T - \rho C_e(\dot{T} + t_0\ddot{T}) - \beta_1 T_0\nabla.(\dot{\vec{u}} + t_0\ddot{\vec{u}}) - aT_0(\dot{C} + t_0\ddot{C}) = 0 \tag{2}$$

$$\nabla^2 C - \frac{1}{Db}(\dot{C} + t_1\ddot{C}) - \frac{\beta_2}{b}T_0\nabla^2(\nabla \cdot \vec{u}) - \frac{a}{b}\nabla^2 T = 0 \tag{3}$$

where $\beta_1 = (3\lambda + 2\mu)\alpha_T$; $\beta_2 = (3\lambda + 2\mu)\alpha_C$; α_T and α_C are the coefficients of linear thermal expansion and linear diffusion expansion; λ and μ are Lame's parameters; ρ, C_e, and K are, respectively, the density, specific heat at constant strain and thermal conductivity; and a and b are thermo-diffusive and diffusive constants, respectively. Here, t_0 and t_1 are thermal relaxation times.

We define the quantities

$$\begin{gathered} x' = \frac{\omega^* x}{C_L}, \quad z' = \frac{\omega^* z}{C_L}, \quad t' = \omega^* t, \quad u' = \frac{\rho\omega^* C_L u}{\beta_1 T_0}, \quad w' = \frac{\rho\omega^* C_L w}{\beta_1 T_0}, \quad T' = \frac{T}{T_0}, \quad C' = \frac{C}{C_0} \\ \omega^* = \frac{C_e(\lambda + 2\mu)}{K}, \quad \varepsilon_T = \frac{T_0\beta_1^2}{\rho C_e(\lambda + 2\mu)}, \quad \delta^2 = \frac{C_S^2}{C_L^2}, \quad \bar{\beta} = \frac{\beta_2 C_0}{\beta_1 T_0}, \quad \varepsilon_c = \frac{\beta_1\beta_2 T_0}{C_0 b\,(\lambda + 2\mu)} \\ C_L^2 = \frac{\lambda + 2\mu}{\rho}, \quad C_S^2 = \frac{\mu}{\rho}, \quad t_0' = \omega^* t_0, \quad t_1' = \omega^* t_1, \quad \bar{a} = \frac{aC_0}{\rho C_e}, \quad \bar{b} = \frac{aT_{\bar{0}}}{bC_0}, \quad \bar{\omega}_b = \frac{C_L^2}{\omega^* Db}. \end{gathered} \tag{4}$$

On introducing quantities (4) in Eqs. (1)–(3), we obtain

$$\delta^2\nabla^2\vec{u} + (1 - \delta^2)\nabla(\nabla \cdot \vec{u}) - \nabla T - \bar{\beta}\nabla C = \ddot{\vec{u}} \tag{5}$$

$$\nabla^2 C - \bar{\omega}_b(\dot{C} + t_1\ddot{C}) - \varepsilon_C\nabla^2(\nabla \cdot \vec{u}) - \bar{b}\nabla^2 T = 0 \tag{6}$$

$$\nabla^2 T - (\dot{T} + t_0\ddot{T}) - \varepsilon_T\nabla \cdot (\dot{\vec{u}} + t_0\ddot{\vec{u}}) - \bar{a}(\dot{C} + t_0\ddot{C}) = 0 \tag{7}$$

where $\nabla = (\partial/\partial x, 0, \partial/\partial z)$ and $\nabla^2 = (\partial^2/\partial^2 x) + (\partial^2/\partial^2 z)$ are the gradient and Laplacian operators. Here, primes have been suppressed for convenience.

3. Boundary conditions

The non-dimensional boundary conditions to be satisfied at the surface $z = 0$ are as follows:

(i) The normal components of stress tensor must vanish at the surface, which implies that

$$(1 - 2\delta^2)\nabla \cdot \vec{u} + 2\delta^2 \frac{\partial w}{\partial z} - T - \bar{\beta}C = 0. \tag{8}$$

(ii) The tangential components of stress tensor also must vanish. This leads to

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0. \tag{9}$$

(iii) Mass concentration must vanish at the surface so that we have

$$C = 0. \tag{10}$$

(iv) The thermal boundary conditions on the surface are given by

$$\frac{\partial T}{\partial z} + hT = 0 \tag{11}$$

where h is the surface heat transfer coefficient. Here, ($h \to 0$) corresponds to the thermally insulated boundary and ($h \to \infty$) refers to the isothermal one.

4. Solution of the problem

In order to solve the problem we introduce the potential functions φ and ψ through the relations

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \varphi}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{12}$$

On using relations (12) in Eqs. (5)–(7), we obtain

$$\nabla^2 \varphi - \ddot{\varphi} - \bar{\beta}C - T = 0 \tag{13}$$

$$\nabla^2 \psi = \frac{\ddot{\psi}}{\delta^2} \tag{14}$$

$$\nabla^2 T - (\dot{T} + t_0 \ddot{T}) - \varepsilon_T(\dot{\varphi} + t_0 \ddot{\varphi}) - \bar{a}(\dot{C} + t_0 \ddot{C}) = 0 \tag{15}$$

$$\nabla^2 C - \bar{\omega}_b(\dot{C} + t_1 \ddot{C}) - \nabla^4 \varphi - \bar{b}\nabla^2 T = 0. \tag{16}$$

It is observed that the transverse motion represented by the function ψ gets decoupled from the rest of the motion corresponding to the functions ψ, C, and T. We take the solution of the form

$$(\varphi, \psi, C, T) = (\tilde{\varphi}(z), \tilde{\psi}(z), \tilde{C}(z), \tilde{T}(z))e^{i\xi(x-ct)} \tag{17}$$

where $c = \omega/\xi$ is non-dimensional phase velocity, ω and ξ are the non-dimensional circular frequency and wavenumber, respectively. On using solution (17) in Eqs. (13)–(16) and solving the resulting system of equations, the expressions for φ, ψ, T, and C are obtained as

$$\varphi = \left[\sum_{j=1}^{3}(A_J e^{m_j z} + B_J e^{-m_j z})\right] e^{i\xi(x-ct)} \tag{18}$$

$$T = \left[\sum_{j=1}^{3} S_j (A_J e^{m_j z} + B_J e^{-m_j z})\right] e^{i\xi(x-ct)} \tag{19}$$

$$C = \left[\sum_{j=1}^{3} V_j (A_J e^{m_j z} + B_J e^{-m_j z})\right] e^{i\xi(x-ct)} \tag{20}$$

$$\psi = (A_4 e^{\beta z} + B_4 e^{-\beta z}) e^{i\xi(x-ct)} \tag{21}$$

$$S_j = \frac{\omega^2[(1-a_j^2)\{(1+\bar{\beta}\bar{b})a_j^2 - \tau_1\bar{\omega}_b\} - \bar{\beta}\bar{b}a_j^2\{1-(1+\varepsilon_a)a_j^2\}]}{(1+\bar{\beta}\bar{b})a_j^2 - \tau_1\bar{\omega}_b} \tag{22a}$$

$$V_j = \frac{\omega^2\bar{b}a_j^2[1-(1+\varepsilon_a)a_j^2]}{(1+\bar{\beta}\bar{b})a_j^2 - \tau_1\bar{\omega}_b} \tag{22b}$$

where $m_j^2 = \xi^2(1-a_j^2c^2), j = 1,2,3,\ \alpha^2 = \xi^2(1-c^2)$

$$\beta^2 = \xi^2\left(1-\frac{c^2}{\delta^2}\right), \quad \tau_1 = t_1 + i\omega^{-1}, \quad \tau_0 = t_0 + i\omega^{-1}, \quad \varepsilon_a = \frac{\varepsilon_L}{\bar{b}} \tag{23}$$

Here $a_j^2, j = 1,2,3$ are roots of complex cubic equation

$$\zeta^3 - L\zeta^2 + M\zeta - N = 0 \tag{24}$$

where

$$L = \frac{[1+\tau_1\bar{\omega}_b + \tau_0\{(1+\bar{a}\bar{b})(1+\varepsilon_a) + (1+\bar{\beta}\bar{b})(\varepsilon_T - \varepsilon_a)\}]}{(1-\varepsilon_c\bar{\beta})} \tag{25}$$

$$M = \frac{\tau_0(1+\bar{a}\bar{b}) + \tau_1\bar{\omega}_b\{1+\tau_0(1+\varepsilon_T)\}}{(1-\varepsilon_c\bar{\beta})} \tag{26}$$

$$N = \frac{\tau_0\tau_1\bar{\omega}_b}{(1-\varepsilon_c\bar{\beta})}. \tag{27}$$

As we are interested in the study of surface waves so the disturbance is assumed to be confined to the boundary $z = 0$ of the half-space. Therefore, we select the form of solutions (18)–(21) that satisfies the radiation condition namely, $\mathrm{Re}(m_j, \beta) \geqslant 0, j = 1,2,3$. Thus, the required expressions for φ, T, C, and ψ are written as

$$\varphi = \sum_{j=1}^{3} B_j e^{-m_j z + i\xi(x-ct)} \tag{28}$$

$$T = \sum_{j=1}^{3} S_j B_j e^{-m_j z + i\xi(x-ct)} \tag{29}$$

$$C = \sum_{j=1}^{3} V_j B_j e^{-m_j z + i\xi(x-ct)} \tag{30}$$

$$\psi = B_4 e^{-\beta z + i\xi(x-ct)}. \tag{31}$$

The displacement components u and w are obtained from Eqs. (12) on using solutions (28) and (31) we obtain

$$u = \left(\mathrm{i}\xi \sum_{j=1}^{3} B_j \mathrm{e}^{-m_j z} - \beta B_4 \mathrm{e}^{-\beta z} \right) \mathrm{e}^{\mathrm{i}\xi(x-ct)} \tag{32}$$

$$w = -\left(\sum_{j=1}^{3} m_j B_j \mathrm{e}^{-m_j z} + \mathrm{i}\xi B_4 \mathrm{e}^{-\beta z} \right) x \mathrm{e}^{\mathrm{i}\xi(x-ct)}. \tag{33}$$

The stresses can also be obtained in a similar manner.

5. Derivation of the secular equations

Upon invoking the boundary conditions (8)–(11) via relations (12) at the surface $z = 0$ and using Eqs. (28)–(31), we obtain a system of four simultaneous linear equations in the unknown amplitudes B_1, B_2, B_3, and B_4 as follows:

$$p(B_1 + B_2 + B_3) + qB_4 = 0 \tag{34}$$

$$f_1 B_1 + f_2 B_2 + f_3 B_3 + pB_4 = 0 \tag{35}$$

$$V_1 B_1 + V_2 B_2 + V_3 B_3 = 0 \tag{36}$$

$$S_1(h - m_1)B_1 + S_2(h - m_2)B_2 + S_3(h - m_3)B_3 = 0, \tag{37}$$

where $p = \beta^2 + \xi^2$, $q = 2\mathrm{i}\xi\beta$, $f_j = 2\mathrm{i}\xi m_j$, $j = 1,2,3$.

The system of Eqs. (34)–(37) has a non-trivial solution if and only if the determinant of the coefficients of $(B_1, B_2, B_3, B_4)^{\mathrm{T}}$ vanishes. After applying lengthy algebraic reductions and manipulations, this leads to the secular equations for the surface waves in the considered medium. We obtain

$$(\beta^2 + \xi^2)^2 F + 4\xi^2 \beta G = h\{(\beta^2 + \xi^2)^2 F^* + 4\xi^2 \beta G^*\} \tag{38}$$

where

$$F = m_2 S_2(V_1 - V_3) + m_3 S_3(V_2 - V_1) + m_1 S_1(V_3 - V_1) \tag{39a}$$

$$G = m_1 m_2 V_3(S_1 - S_2) + m_2 m_3 V_1(S_2 - S_3) + m_1 m_3 V_2(S_3 - S_1) \tag{39b}$$

$$F^* = (V_1 - V_3)(S_2 - S_3) - (V_2 - V_3)(S_1 - S_3) \tag{40a}$$

$$G^* = m_1(V_2 S_3 - V_3 S_2) + m_2(V_3 S_1 - V_1 S_3) + m_3(V_1 S_2 - V_2 S_1). \tag{40b}$$

Eq. (38) is the required combined secular equation, which governs the propagation Rayleigh-type surface waves in the considered media. For thermally insulated boundary ($h \to 0$) the secular equation (38) becomes

$$(\beta^2 + \xi^2)^2 F + 4\xi^2 \beta G = 0 \tag{41}$$

and in case of isothermal boundary ($h \to \infty$) it reduces to

$$(\beta^2 + \xi^2)^2 F^* + 4\xi^2 \beta G^* = 0 \tag{42}$$

where F, G, F^*, and G^* are defined in Eqs. (39) and (40).

Eqs. (41) and (42) are the secular equations for Rayleigh surface waves in a half-space subjected to stress free, isoconcentrated, thermally insulated and stress free, isoconcentrated, isothermal boundary conditions, respectively. These frequency equations contain complete information regarding wavenumber, frequency and phase velocity of the waves.

6. Generalized thermoelastic waves

In the absence of MD ($a = 0 = \beta_2$), the secular equation (38) takes the form

$$(\beta^2 + \xi^2)^2(m_1^2 + m_1 m_3 + m_3^2 - \alpha^2) + 4\xi^2\beta m_1 m_3(m_1 + m_3) = h\{(m_1 + m_3)(\beta^2 + \xi^2)^2 + 4\xi^2\beta(m_1 m_3 + \alpha^2)\}. \tag{43}$$

This leads to

$$(\beta^2 + \xi^2)^2(m_1^2 + m_1 m_3 + m_3^2 - \alpha^2) + 4\xi^2\beta m_1 m_3(m_1 + m_3) = 0 \tag{44}$$

for thermally insulated ($h \to 0$) boundary of the half-space and

$$(m_1 + m_3)(\beta^2 + \xi^2)^2 + 4\xi^2\beta(m_1 m_3 + \alpha^2) = 0 \tag{45}$$

in case of the isothermal ($h \to \infty$) one.

Eqs. (44) and (45) are the secular equations that govern generalized thermo-elastic Rayleigh waves. These equations are the same as obtained and discussed in detail by Nayfeh and Nasser [13] and also reported by Dhaliwal and Singh [21]. The complex secular equation (38) and hence Eqs. (41) and (42) contain complete information regarding phase velocity, attenuation coefficient and wavenumber of the surface waves in the considered environment.

Owing to the complex nature of these secular equations, the development of their analytical solution is a cumbersome exercise. However, approximate and numerical techniques can be conveniently used to explore various features of the waves. Therefore, here we have used irreducible Cardano's method with the help of DeMoivre's theorem to obtain the complex characteristic roots $a_j^2, j = 1, 2, 3$ of Eq. (24) and hence compute complex roots α^2, β^2, and $m_j^2, j = 1, 2, 3$ given by Eq. (23). The secular equations (41) and (42) are then solved for phase velocity and attenuation coefficient by using the functional iteration technique of numerical analysis through the relation

$$C^{-1} = V^{-1} + \mathrm{i}\omega^{-1}Q \tag{46}$$

where V, Q, ω are real numbers and

$$\xi = R + \mathrm{i}Q, \quad R = \frac{\omega}{V}.$$

The specific loss factor being the measure of energy dissipation in a specimen through a stress cycle (ΔW) to the elastic energy (W) stored in the specimen at maximum strain is also computed. For a sinusoidal plane wave of small amplitude, Kolsky [20] shows that the specific loss $\Delta W/W$ equals 4π times the absolute value of the imaginary part of ξ to the real part of ξ.

Hence,

$$\left|\frac{\Delta W}{W}\right| = 4\pi\left|\frac{\mathrm{Im}(\xi)}{\mathrm{Re}(\xi)}\right| = 4\pi\left|\frac{VQ}{W}\right|. \tag{47}$$

The thermo-mechanical coupling factor (K^2) is defined as

$$K^2 = \left|\frac{V_{\text{ins}} - V_{\text{iso}}}{V_{\text{iso}}}\right| \tag{48}$$

where V_{ins} and V_{iso} are the real phase speeds of the wave under thermally insulated and isothermal boundary conditions prevailing at the stress-free surface of the material half-space.

7. Surface displacements, temperature change, and mass concentration

In this section, we derive expressions for surface displacements, temperature change, and mass concentration in addition to the discussion of motion of surface particles. At the surface $z = 0$ of the half-space, the displacements, temperature, and mass concentration during the Rayleigh surface wave propagation are obtained as

$$u_S = |X|A\mathrm{e}^{\mathrm{i}(P-\theta_1)} \tag{49}$$

$$w_S = |Y|Ae^{i(P-\theta_2)} \tag{50}$$

$$T_S = |\Theta|Ae^{i(P-\theta_3)} \tag{51}$$

$$C_S = |Z|Ae^{i(P-\theta_4)} \tag{52}$$

where

$$\begin{aligned} &X = i\xi(1 + L_1 + M_1) + \beta N_1; \quad \theta_1 = \mathrm{Arg}(X) \\ &Y = -(m_1 + m_2 L_1 + m_3 M_1 + i\xi N_1); \quad \theta_2 = \mathrm{Arg}(Y) \\ &\Theta = S_1 + S_2 L_1 + S_3 M_1; \quad \theta_3 = \mathrm{Arg}(\Theta) \\ &Z = V_1 + V_2 L_1 + V_3 M_1; \quad \theta_4 = \mathrm{Arg}(Z) \\ &A = B_1 e^{-Qx}; \quad P = R(x - Vt) \end{aligned} \tag{53}$$

$$\begin{aligned} L_1 &= \frac{\{4\xi^2\beta m_3 + (\beta^2+\xi^2)^2\}V_1 - \{4\xi^2\beta m_1 + (\beta^2+\xi^2)^2\}V_3}{\{4\xi^2\beta m_2 + (\beta^2+\xi^2)^2\}V_3 - \{4\xi^2\beta m_3 + (\beta^2+\xi^2)^2\}V_2} \\ M_1 &= \frac{\{4\xi^2\beta m_1 + (\beta^2+\xi^2)^2\}V_2 - \{4\xi^2\beta m_2 + (\beta^2+\xi^2)^2\}V_1}{\{4\xi^2\beta m_2 + (\beta^2+\xi^2)^2\}V_3 - \{4\xi^2\beta m_3 + (\beta^2+\xi^2)^2\}V_2} \\ N_1 &= \frac{2i\xi(\beta^2+\xi^2)^2\{(m_2-m_1)(V_3-V_1) - (m_3-m_1)(V_2-V_1)\}}{\{4\xi^2\beta m_2 + (\beta^2+\xi^2)^2\}V_3 - \{4\xi^2\beta m_3 + (\beta^2+\xi^2)^2\}V_2}. \end{aligned} \tag{54}$$

Thus, there exist phase differences between different pairs of quantities u, w, T, and C being complex quantities.

8. Motion of surface particles

Now we shall discuss motion of a typical surface particle during the Rayleigh surface wave propagation. On eliminating P from the surface displacements u_S and w_S in Eqs. (49) and (50), we obtain

$$\frac{u_S^2}{|X|^2} - \frac{2u_S w_S}{|X||Y|}\cos(\theta_1 - \theta_2) + \frac{w_S^2}{|Y|^2} = A^2 \sin^2(\theta_1 - \theta_2). \tag{55}$$

Because

$$\frac{\cos^2(\theta_1-\theta_2)}{|X|^2|Y|^2} - \frac{1}{|X|^2|Y|^2} = \frac{-\sin^2(\theta_1-\theta_2)}{|X|^2|Y|^2} < 0$$

so Eq. (55) represents an ellipse with semimajor axis (a^*), semiminor axis (b^*) and eccentricity (e), given by

$$a^{*^2} = \frac{A^2}{2}\left[|X|^2 + |Y|^2 + \sqrt{\left(|X|^2 - |Y|^2\right)^2 + 4|X|^2|Y|^2\cos^2(\theta_1-\theta_2)}\right] \tag{56}$$

$$b^{*^2} = \frac{A^2}{2}\left[|X|^2 + |Y|^2 - \sqrt{\left(|X|^2 - |Y|^2\right)^2 + 4|X|^2|Y|^2\cos^2(\theta_1-\theta_2)}\right] \tag{57}$$

$$e^2 = \frac{2\sqrt{\left(|X|^2 - |Y|^2\right)^2 + 4|X|^2|Y|^2\cos^2(\theta_1-\theta_2)}}{|X|^2 + |Y|^2 + \sqrt{\left(|X|^2 - |Y|^2\right)^2 + 4|X|^2|Y|^2\cos^2(\theta_1-\theta_2)}}. \tag{58}$$

The inclination (δ^*) of wave normal with the major axis of the elliptical path of a typical particle is also computed and is obtained as

$$\tan(2\delta^*) = \frac{2\tan\theta(|X|^2 - |Y|^2) + 2|X||Y|\cos(\theta_1-\theta_2)(1-\tan^2\theta)}{(|X|^2 - |Y|^2)(1-\tan^2\theta) - 4|X||Y|\cos(\theta_1-\theta_2)\tan\theta} \tag{59}$$

where θ is the inclination of wave normal with the z-axis. As we are dealing with Rayleigh surface waves, so usually $\theta = \pi/2$ and hence we obtain

$$\delta^* = \frac{1}{2}\tan^{-1}\left(\frac{2|X||Y|\cos(\theta_1 - \theta_2)}{|X|^2 - |Y|^2}\right). \tag{60}$$

Clearly, the particle paths becomes linear when there is no phase difference between the functions u_S and w_S.

9. Numerical result and discussion

In order to illustrate and verify the analytical results obtained in the previous sections, we present some numerical simulation results. The materials chosen for this purpose are copper (solvent) and zinc (solute), whose physical data are given as follows:

$$\lambda = 8.2\times10^{10}\,\mathrm{N\,m^{-2}},\quad \mu = 4.2\times10^{10}\,\mathrm{N\,m^{-2}},\quad \rho = 8.950\times10^{3}\,\mathrm{kg\,m^{-3}}$$
$$T_0 = 300^\circ\,\mathrm{K},\quad C_e = 0.8298\times10^{-3}\,\mathrm{J\,kg^{-1}\,K^{-1}},\quad K = 1.13\times10^{2}\,\mathrm{W\,m^{-1}\,s^{-1}\,K^{-1}}$$
$$\alpha_T = 1.0\times10^{-8}\,\mathrm{K^{-1}},\quad D = 0.34\times10^{-4}\,\mathrm{m\,s^{-1}}(\mathrm{Zn{-}Cu}),\quad \varepsilon_T = 0.00265$$
$$\beta_1 = 3300\,\mathrm{N\,m^{-2}K^{-1}},\quad \beta_2 = 330\,\mathrm{N\,m^{-5}K^{-1}},\quad \omega^* = 1.11\times10^{11}\,\mathrm{s^{-1}}$$
$$\alpha_c = 1.0\times10^{-9}\,\mathrm{K^{-1}},\quad a = 0.1521\times10^{2}\,\mathrm{m\,s^{-1}},\quad b = 0.02\times10^{-4}\,\mathrm{m\,s^{-1}}.$$

After computing the complex roots given by Eq. (24) with the help of reduced Cardano's method and using these in various relevant relations, the secular equations (41) and (42) are then solved to obtain the phase velocity and attenuation coefficient by using the iteration method. The phase velocity and attenuation coefficient have been computed for various values of the real wavenumber from secular equations (41) and (42) for insulated and equipotential boundary conditions of thermal and concentration fields by developing a FORTRAN program on an IBM PENTIUM-IV computer. In Figs. 1–5, the solid curves correspond to the stress free, isothermal, and isoconcentrated boundary of the half-space and dotted curves refer to the stress free, insulated boundary conditions of the considered half-space. In Fig. 6, the solid curve corresponds to coupled thermoelasticity and the dotted to that of the generalized theory of thermoelasticity.

In Fig. 1, the phase velocity is plotted with respect to wavenumber on the semilogarithmic scale. It is observed that the phase velocity decreases sharply from a high value at vanishing wavenumber in the range $0 \leqslant R \leqslant 1$, it attains minimum value in the region $1 \leqslant R \leqslant 2$ and becomes almost constant for $R \geqslant 2$, in case of both isothermal and thermally insulated boundaries of the half-space. The value of phase velocity in case of isothermal boundary of the half-space is observed to be significantly large as compared with the thermally insulated one in the wavenumber range $0 \leqslant R \leqslant 1$. Moreover, negligibly small effects of relaxation time are observed on the phase velocity, in both the considered cases, because the two curves for ($\tau_0 = 0 = \tau_1$) and

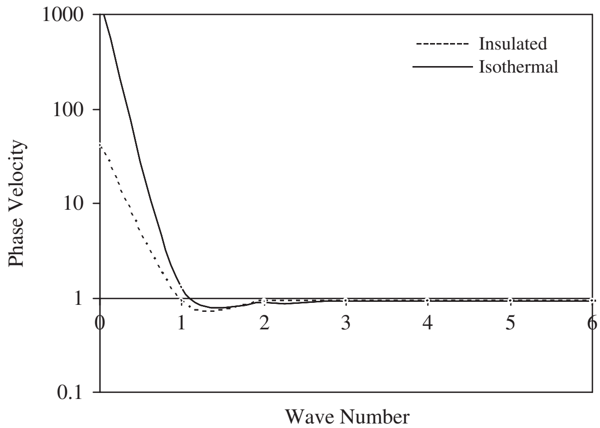

Fig. 1.

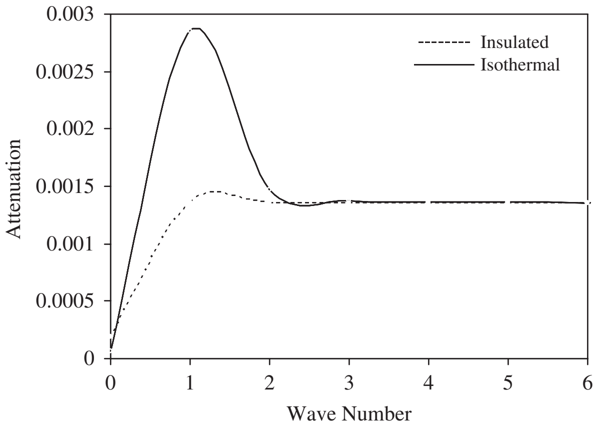

Fig. 2.

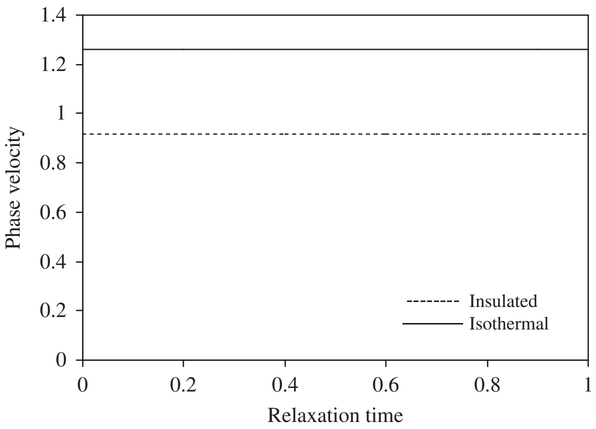

Fig. 3.

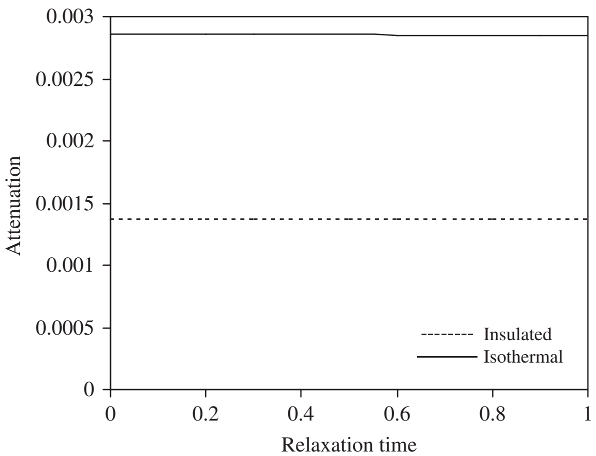

Fig. 4.

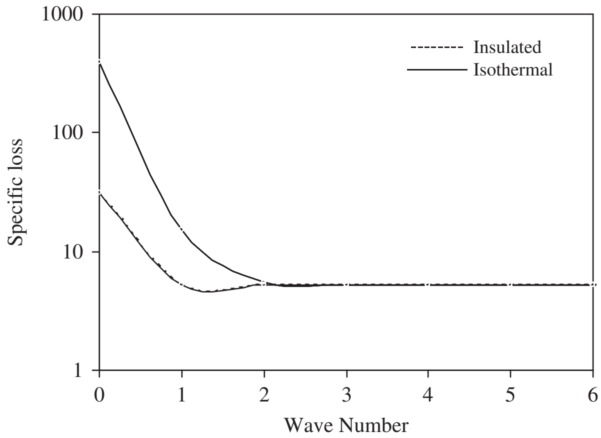

Fig. 5.

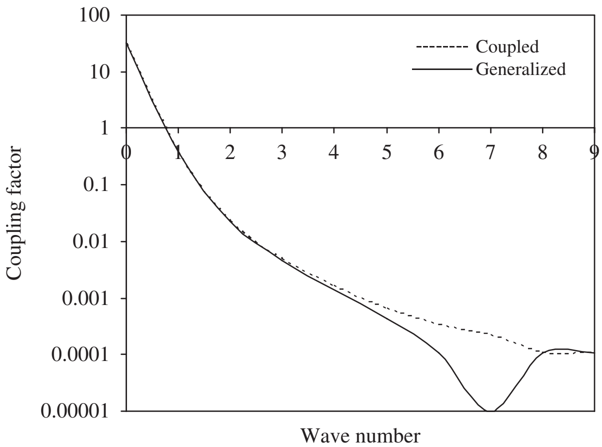

Fig. 6.

($\tau_0 = 0.5, \tau_1 = 0.4$) almost overlap. From Fig. 2, it is observed that the attenuation coefficient for both cases of boundary conditions increases from a value very near to zero at vanishing wavenumber in the range $0 \leqslant R \leqslant 1$ and then decreases for $1 \leqslant R \leqslant 2$, before becoming steady and stable afterwards for $R \geqslant 2$. The variation in the magnitude of attenuation is quite dispersive in the range $0 \leqslant R \leqslant 2$ for isothermal and isoconcentrated boundary conditions prevailing at the surface of the half-space as compared with the insulated one. From Figs. 3 and 4, it is noticed that the phase velocity and attenuation coefficient are constant with respect to relaxation time for both the considered cases of boundary conditions of the half-space. The magnitudes of phase velocity and attenuation coefficient are higher for isothermal as compared with thermally insulated boundary of the half-space. This shows that thermal relaxation time has significantly large effects on the considered quantities at isothermal conditions as compared with that in of the insulated one.

It is revealed from Fig. 5 that the specific loss factor of energy dissipation, a measure of internal friction of the material (mapped on semilogarithmic scale), decreases from a highest value at vanishing wavenumber in the wavenumber range $0 \leqslant R \leqslant 1$ for insulated conditions and $0 \leqslant R \leqslant 2$ for the isothermal one before becoming stable and steady for $R \geqslant 2$ in both cases. In the wavenumber range $0 \leqslant R \leqslant 2$, the specific loss profiles in case of insulated and isothermal boundary conditions are at a significant departure from each other, although both are dispersive in character. This clearly depicts the effect of the boundaries on the propagation characteristics of the surface waves in the considered environment.

Fig. 6 shows the variations of thermo-mechanical coupling factor (K^2) in coupled ($\tau_0 = 0 = \tau_1$) and generalized ($\tau_0 = 0.5, \tau_1 = 0.4$) theories of thermoelasticity with wavenumber. In both cases, the magnitude of the thermo-mechanical factor is large at vanishing wavenumber, which decreases sharply in the wave range $0 \leqslant R \leqslant 7$. For coupled theory, it becomes zero at $R = 7$ and then increases in the range $7 < R < 8$. The wavenumber varyies steadily afterwards for $R \geqslant 8$, for both cases. This clearly depicts the effect of thermo-mechanical coupling among various interacting fields. This phenomenon is quite physically realistic because for long wave length waves the effect of coupling among various considered field quantities is quite predominant because of their deep penetration into the medium. The coupling among various interacting fields ceases to zero in case of short wave length waves because they mainly travel along the surface without much penetration into the medium thereby less disturbance to the interacting fields.

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